Symmetry and reciprocity constraints on diffraction by gratings of quasi-planar particles

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Abstract

Symmetry and reciprocity constraints on polarization state of the field diffracted by gratings of quasi-planar particles are considered. It is shown that the optical activity effects observed recently in arrays of quasi-planar plasmonic particles on a dielectric substrate are due to the reflection of the field at the air-dielectric slab interface and are proportional to this reflection coefficient.

1 Introduction

Two-dimensional periodical grids of thin planar metal particles of various complex shapes show interesting electromagnetic properties and find applications in radio and microwave engineering as frequency selective surfaces and polarization transformers (e.g., [1]). With recent advances in nanofabrication, optical properties of such structures start attracting considerable interest of researchers. In particular interest are 2D-chiral inclusions, that do not possess mirror symmetry under two-dimensional reflections with respect to a line in the particle plane. Various gammadion shapes have been recently studied theoretically and experimentally in the optical region, e.g., [2, 3, 4].

The symmetry of these shapes has an interesting analogy with magnetized ferrites and with three-dimensional (volumetric) chiral particles. The gammadion (swastika) geometry defines an axial unit vector normal to the particle plane, whose direction is determined by the tilt direction of the particle arms and the right-hand rule. Similarly, in the case of a magnetized ferrite sample, there is a selected direction defined by the bias magnetic field. At first sight, this suggests that in arrays of 2D-chiral particles there can be electromagnetic phenomena analogous to the Faraday rotation of the polarization plane. And indeed, polarization rotation (optical activity) has been observed in arrays of swastika-shaped particles on

a dielectric [2, 3, 4]. However, in [3] this phenomenon was mistakenly attributed to broken time-reversal symmetry or nonreciprocity (note that the Faraday effect is nonreciprocal). Later it was experimentally shown that the rotation of the polarization plane of transmitted waves was reciprocal [4]. Moreover, in [4] the dielectric substrate was found to be the key factor for the optical activity in quasi-planar arrays of gammadions because adding a substrate would effectively turn an array of 2D-chiral objects into a 3D-chiral structure which would exhibit the usual reciprocal optical activity.

Despite apparent similarity, polarization rotation phenomena are fundamentally different in reciprocal and nonreciprocal systems. In the case of magnetized ferrites, time reversal operation reverses the direction of the axial vector defining the bias field direction, while in the case of arrays of reciprocal particles, time reversal does not affect the direction of the axial vector showing the particle “rotation sense”. Any system formed by metal and dielectric inclusions of any shapes is reciprocal, because there is no time-odd physical parameter there and the Maxwell equations themselves are invariant under time reversal operation.

It is well known that polarization plane rotation exists also in reciprocal media. Optical activity is the main manifestation of three-dimensional chirality of molecules or artificial inclusions forming chiral and, more generally, bi-anisotropic materials [5, 6]. This effect is governed by the chirality parameter (the trace of the magnetoelectric coupling dyadic), which vanishes for planar particles of arbitrary shapes, because any 2D shape can always be superimposed with its mirror image if rotations in 3D space are allowed. In fact, the magnetoelectric coupling dyadic of planar particles is antisymmetric, and composites formed by such particles belong to the class of omega media [6]. This apparently imposes restrictions on optical activity effects in arrays of 2D particles, even if their shape is chiral in the two-dimensional space.

Electromagnetic properties of 2D-chiral arrays in the microwave region were studied earlier (e.g. [7, 8]) and these structures were even patented [9]. However, to the best of our knowledge, no systematic consideration of fundamental limitations on electromagnetic scattering from general grids of quasi-planar particles has been made. Important results were obtained in [7, 10, 11] from the symmetry considerations, but those results concern restrictions on the form of the constitutive tensors of composite media but not the diffraction phenomena. The theory of [12] concerns only arrays in free space and does not include the effects of the substrate.

In this paper we make a systematic study of reciprocity and symmetry restrictions on plane-wave diffraction by 2D arrays of arbitrary shaped quasi-planar particles. By quasi-planar particles we understand particles of a finite thickness but with uniform properties over the full particle thickness. The arrays can be positioned on planar dielectric substrates, and we will see that the presence of the substrate is of key importance for optical activity phenomena in these structures.

2 Problem formulation

We consider an infinite two-dimensional array of arbitrary shaped (in the array plane) particles backed with a slab of a homogeneous dielectric. The thickness of the particles is assumed to be finite, but the structure is uniform over its thickness. This implies that the array has

a plane of symmetry that itself can be regarded as the array plane. In practice, the particles are usually made of metal, but the theory which we develop here applies to any reciprocal isotropic material. In case of metal inclusions, the general theory developed here applies both to microwave frequencies and to the visible range, because particular electromagnetic parameters of metal particles do not affect restrictions which follow from symmetry and reciprocity considerations. We assume that the array particles are not touching the dielectric substrate, while the separation of the dielectric and the particles can be arbitrary small. This assumption allows us to split the entire structure in two parts: the grating and the slab and consider them separately. An example of the problem geometry is illustrated in Fig. 1.

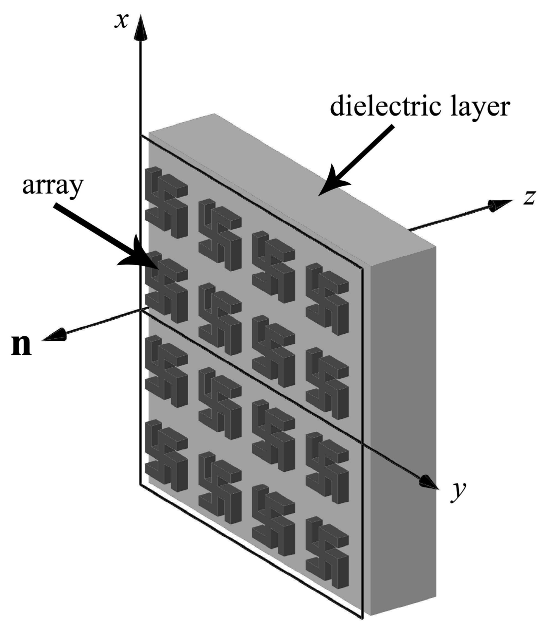

Figure 1: An array of metal (plasmonic) particles backed by a dielectric slab. **n** is the unit vector normal to the array plane.

The array is illuminated by a plane wave of arbitrary polarization and arbitrary propagation direction[1] incoming from the half space $z < 0$. We are interested in the polarization states of the transmitted and reflected waves[2] and how it relates to the polarization of the incident wave. We will attack this problem with a rather general approach taking into account symmetry and reciprocity restrictions.

3 Symmetry and reciprocity restrictions

Let us start from considering the array of particles separately from the dielectric substrate. Under our assumptions there exist two planes in space which enclose the grating entirely

[1]Evanescent-wave excitation is not excluded from this analysis.

[2]The case of multiple propagating modes created by the array is not excluded: In that case the following restrictions apply to every excited mode.

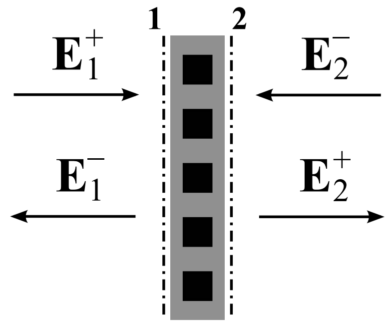

Figure 2: Definitions of the incident and scattered waves for a quasi-planar grid. The arrows show the normal to the interfaces components of the wave vector.

in between themselves. We split the tangential electric fields on these two planes into the incident fields exciting the grating and the fields scattered by the grating (see Fig. 2) and introduce the following scattering matrix:

$$\begin{pmatrix} \mathbf{E}_1^-(\mathbf{r}) \\ \mathbf{E}_2^+(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \widehat{S}_{11} & \widehat{S}_{12} \\ \widehat{S}_{21} & \widehat{S}_{22} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{E}_1^+(\mathbf{r}) \\ \mathbf{E}_2^-(\mathbf{r}) \end{pmatrix}. \tag{1}$$

Here $\mathbf{E}_1^+(\mathbf{r})$ and $\mathbf{E}_2^-(\mathbf{r})$ denote the vectorial complex amplitudes of the tangential components of the incident electric field on the first and the second surface, respectively. $\mathbf{E}_1^-(\mathbf{r})$ and $\mathbf{E}_2^+(\mathbf{r})$ are the respective scattered fields, and $\widehat{S}_{mn}$ are scattering operators that connect the tangential components of the scattered and incident electric fields at surfaces 1 and 2.

The fields at the two surfaces can be expanded into plane waves

$$\mathbf{E}(\mathbf{k}_\mathrm{t}) = \iint \mathbf{E}(\mathbf{r})\, e^{-i\mathbf{k}_\mathrm{t}\cdot\mathbf{r}} d^2\mathbf{r}, \tag{2}$$

where the integration is taken over the surface plane, $\mathbf{E}$ can be any of the fields defined above, and $\mathbf{k}_\mathrm{t}$ is the wave vector component tangential to the planes. We use the time dependence of the form $\exp(-i\omega t)$ conventionally used in optics.

This expansion gives us a representation of the operators $\widehat{S}_{mn}$ in the basis of plane waves. The corresponding matrix elements of the operators $\widehat{S}_{mn}$ in this basis are dyadics $\overline{\overline{s}}_{mn}(\mathbf{p},\mathbf{q})$:

$$\overline{\overline{s}}_{mn}(\mathbf{p},\mathbf{q}) = \iint e^{-i\mathbf{p}\cdot\mathbf{r}}\, \widehat{S}_{mn}\, e^{i\mathbf{q}\cdot\mathbf{r}} d^2\mathbf{r}. \tag{3}$$

If the grating has a center of symmetry and the planes where we define the fields are located at symmetric positions with respect to the same center, the dyadics $\overline{\overline{s}}_{mn}(\mathbf{p},\mathbf{q})$ obey the following restrictions:

$$\overline{\overline{s}}_{11}(\mathbf{p},\mathbf{q}) = \overline{\overline{s}}_{22}(-\mathbf{p},-\mathbf{q}), \quad \overline{\overline{s}}_{12}(\mathbf{p},\mathbf{q}) = \overline{\overline{s}}_{21}(-\mathbf{p},-\mathbf{q}). \tag{4}$$

Indeed, applying the coordinate transform $\mathbf{r} \to -\mathbf{r}$ where the origin is at the symmetry center, we come to the same structure with the first and the second interfaces interchanged. On the other hand, the components of polar vectors change signs under this transformation, and we obtain (4).

If the grating plane is itself a plane of symmetry of the system, then, more obviously,

$$\widehat{S}_{11} = \widehat{S}_{22}, \quad \widehat{S}_{12} = \widehat{S}_{21}, \tag{5}$$

or, which is the same,

$$\overline{\overline{s}}_{11}(\mathbf{p},\mathbf{q}) = \overline{\overline{s}}_{22}(\mathbf{p},\mathbf{q}), \quad \overline{\overline{s}}_{12}(\mathbf{p},\mathbf{q}) = \overline{\overline{s}}_{21}(\mathbf{p},\mathbf{q}). \tag{6}$$

To get this result, one has to apply a transform that changes the sign of the radius vector component normal to the grating plane. Such a transformation interchanges the interfaces as above, but preserves the signs of the tangential components of polar vectors.

Furthermore, if the grating has rotational symmetry of order $m \geq 2$ with respect to an axis parallel to $\mathbf{n}$, the following additional relation holds:

$$\overline{\overline{s}}_{ij}(\overline{\overline{\xi}}\cdot\mathbf{p}, \overline{\overline{\xi}}\cdot\mathbf{q}) = \overline{\overline{\xi}}\cdot\overline{\overline{s}}_{ij}(\mathbf{p},\mathbf{q})\cdot\overline{\overline{\xi}}^{-1}. \tag{7}$$

where $\overline{\overline{\xi}} = \exp[(2\pi/m)[\mathbf{n}\times\overline{\overline{I}}_{\rm t}]]$ is the dyadic of rotation in the grating plane, $\overline{\overline{I}}_{\rm t}$ is the unit dyadic in the same plane.

In a reciprocal system the reciprocity theorem holds. To apply this theorem we assume that the external fields are created by sheets of surface electric currents $\mathbf{J}_{1,2}$ placed at the first and the second interfaces, respectively. Then, at the interfaces

$$\mathbf{E}_1^+(\mathbf{r}) = -\frac{\widehat{Z}_{\rm w}\cdot\mathbf{J}_1(\mathbf{r})}{2}, \quad \mathbf{E}_2^-(\mathbf{r}) = -\frac{\widehat{Z}_{\rm w}\cdot\mathbf{J}_2(\mathbf{r})}{2}, \tag{8}$$

where $\widehat{Z}_{\rm w}$ is the free-space wave impedance operator. In the plane-wave basis it is

$$\overline{\overline{z}}_{\rm w}(\mathbf{p},\mathbf{q}) = (2\pi)^2\,\delta(\mathbf{q}-\mathbf{p})\,\overline{\overline{z}}_{\rm w}(\mathbf{q}), \quad \overline{\overline{z}}_{\rm w}(\mathbf{q}) = \eta\,\frac{\overline{\overline{I}}_{\rm t} - \mathbf{q}\mathbf{q}/k^2}{\sqrt{1-q^2/k^2}}, \tag{9}$$

where $\eta = \sqrt{\mu_0/\varepsilon_0}$, $k = \omega\sqrt{\varepsilon_0\mu_0}$. The operator $\widehat{Z}_{\rm w}$ is diagonal and symmetric as is directly seen from (9):

$$\overline{\overline{z}}_{\rm w}(\mathbf{p},\mathbf{q}) = \overline{\overline{z}}_{\rm w}^{\rm T}(-\mathbf{q},-\mathbf{p}); \quad \overline{\overline{z}}_{\rm w}(\mathbf{p},\mathbf{q}) = 0, \text{ if } \mathbf{p}\neq\mathbf{q}. \tag{10}$$

Because the external field itself satisfies the reciprocity theorem, we can write for the scattered fields

$$\iint \mathbf{E}_{1'}^-\cdot\mathbf{J}_1\,d^2\mathbf{r} + \iint \mathbf{E}_{2'}^+\cdot\mathbf{J}_2\,d^2\mathbf{r} = \iint \mathbf{E}_1^-\cdot\mathbf{J}_{1'}\,d^2\mathbf{r} + \iint \mathbf{E}_2^+\cdot\mathbf{J}_{2'}\,d^2\mathbf{r}. \tag{11}$$

where $\mathbf{J}_{1,1'}$, $\mathbf{J}_{2,2'}$ are the external currents and $\mathbf{E}_{1,1'}^-$, $\mathbf{E}_{2,2'}^+$ are the scattered fields in a pair of separate excitation scenaria.

The currents $\mathbf{J}_{1,2}$, $\mathbf{J}_{1',2'}$ in (11) can be arbitrary chosen and the corresponding electric fields can be expressed from (1) and (8). If, for example, $\mathbf{J}_2 = \mathbf{J}_{2'} = 0$, we get from (1) and (8)

$$\iint \mathbf{J}_1\cdot(\widehat{S}_{11}\cdot\widehat{Z}_{\rm w})\cdot\mathbf{J}_{1'}\,d^2\mathbf{r} = \iint \mathbf{J}_{1'}\cdot(\widehat{S}_{11}\cdot\widehat{Z}_{\rm w})\cdot\mathbf{J}_1\,d^2\mathbf{r}, \tag{12}$$

which in the operator sense means that transpose does not change the operator in braces. Therefore, considering all possible cases, we get for a reciprocal grating:

$$\widehat{S}_{11} \cdot \widehat{Z}_{\rm w} = \widehat{Z}_{\rm w} \cdot \widehat{S}_{11}^{\rm T}, \quad \widehat{S}_{22} \cdot \widehat{Z}_{\rm w} = \widehat{Z}_{\rm w} \cdot \widehat{S}_{22}^{\rm T}, \quad \widehat{S}_{12} \cdot \widehat{Z}_{\rm w} = \widehat{Z}_{\rm w} \cdot \widehat{S}_{21}^{\rm T}. \tag{13}$$

Here we use the fact that $\widehat{Z}_{\rm w} = \widehat{Z}_{\rm w}^{\rm T}$ as given by (10). Relations (13) can be also understood as transpose rules

$$\widehat{S}_{11}^{\rm T} = \widehat{Z}_{\rm w}^{-1} \cdot \widehat{S}_{11} \cdot \widehat{Z}_{\rm w}, \quad \widehat{S}_{22}^{\rm T} = \widehat{Z}_{\rm w}^{-1} \cdot \widehat{S}_{22} \cdot \widehat{Z}_{\rm w}, \quad \widehat{S}_{21}^{\rm T} = \widehat{Z}_{\rm w}^{-1} \cdot \widehat{S}_{12} \cdot \widehat{Z}_{\rm w}. \tag{14}$$

Eq. (14) can be rewritten in terms of the matrix elements of the corresponding operators:

$$\begin{aligned}
\overline{\overline{s}}_{11}^{\rm T}(-\mathbf{q},-\mathbf{p}) &= \overline{\overline{z}}_{\rm w}^{-1}(\mathbf{p}) \cdot \overline{\overline{s}}_{11}(\mathbf{p},\mathbf{q}) \cdot \overline{\overline{z}}_{\rm w}(\mathbf{q}),\\
\overline{\overline{s}}_{22}^{\rm T}(-\mathbf{q},-\mathbf{p}) &= \overline{\overline{z}}_{\rm w}^{-1}(\mathbf{p}) \cdot \overline{\overline{s}}_{22}(\mathbf{p},\mathbf{q}) \cdot \overline{\overline{z}}_{\rm w}(\mathbf{q}),\\
\overline{\overline{s}}_{21}^{\rm T}(-\mathbf{q},-\mathbf{p}) &= \overline{\overline{z}}_{\rm w}^{-1}(\mathbf{p}) \cdot \overline{\overline{s}}_{12}(\mathbf{p},\mathbf{q}) \cdot \overline{\overline{z}}_{\rm w}(\mathbf{q}).
\end{aligned} \tag{15}$$

From (15) one can see that for the waves propagating normally to the grating (when $\mathbf{p} = \mathbf{q} = 0$) reciprocity simply requires

$$\overline{\overline{s}}_{11}(0,0) = \overline{\overline{s}}_{11}^{\rm T}(0,0), \quad \overline{\overline{s}}_{22}(0,0) = \overline{\overline{s}}_{22}^{\rm T}(0,0), \quad \overline{\overline{s}}_{12}(0,0) = \overline{\overline{s}}_{21}^{\rm T}(0,0). \tag{16}$$

Let us note that when compared to the standard mode coupling parameters in the waveguide theory (S-parameters), our scattering dyadics $\overline{\overline{s}}_{mn}(\mathbf{p},\mathbf{q})$ differ in that sense that, first, we do not decompose the fields into TE and TM modes, and, second, we do not separately normalize the modes, instead we deal with a simple plane-wave basis applicable to all types of modes. This allows for a dyadic formalism but results in additional impedance terms in (15).

4 Multiple layers

When there are several layers stacked one on top of another (in our case there are two layers: the grating and the dielectric) the direct application of $\widehat{S}$-matrix (1) becomes tedious. Instead, it is better to apply an approach based on transfer matrices. A transfer matrix connects pairs of fields given at separate interfaces:

$$\begin{pmatrix} \mathbf{E}_1^+(\mathbf{r}) \\ \mathbf{E}_1^-(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} \widehat{T}_{11} & \widehat{T}_{12} \\ \widehat{T}_{21} & \widehat{T}_{22} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{E}_2^+(\mathbf{r}) \\ \mathbf{E}_2^-(\mathbf{r}) \end{pmatrix}. \tag{17}$$

The total transfer matrix of a layered structure is a product of transfer matrices of the layers. The operators $\widehat{T}_{mn}$ can be expressed trough $\widehat{S}_{mn}$ and *vice versa:*

$$\widehat{T}_{11} = \widehat{S}_{21}^{-1}, \quad \widehat{T}_{12} = -\widehat{S}_{21}^{-1} \cdot \widehat{S}_{22}, \quad \widehat{T}_{21} = \widehat{S}_{11} \cdot \widehat{S}_{21}^{-1}, \quad \widehat{T}_{22} = \widehat{S}_{12} - \widehat{S}_{11} \cdot \widehat{S}_{21}^{-1} \cdot \widehat{S}_{22}, \tag{18}$$

$$\widehat{S}_{11} = \widehat{T}_{21} \cdot \widehat{T}_{11}^{-1}, \quad \widehat{S}_{12} = \widehat{T}_{22} - \widehat{T}_{21} \cdot \widehat{T}_{11}^{-1} \cdot \widehat{T}_{12}, \quad \widehat{S}_{21} = \widehat{T}_{11}^{-1}, \quad \widehat{S}_{22} = -\widehat{T}_{11}^{-1} \cdot \widehat{T}_{12}. \tag{19}$$

From these rather complicated relations one can see that the symmetries considered in the previous section generally do not result in simple constraints on $\widehat{T}_{mn}$ (when compared to

$\widehat{S}_{mn}$). This explains why we have started our formulation from the matrix of $\widehat{S}_{mn}$ operators. One exception is the rotational symmetry rule (7) which also holds for $\widehat{T}_{mn}$.

However, there is one important case which results in simple constraints on the elements of the transfer matrix. It is the case when the structure is reciprocal and has a plane of symmetry, i.e. its $\widehat{S}_{mn}$ operators satisfy both (5) and (14). In this case after some operator algebra we obtain

$$\widehat{T}_{11}^{\mathrm{T}} = \widehat{Z}_{\mathrm{w}}^{-1}\cdot\widehat{T}_{11}\cdot\widehat{Z}_{\mathrm{w}}, \quad \widehat{T}_{22}^{\mathrm{T}} = \widehat{Z}_{\mathrm{w}}^{-1}\cdot\widehat{T}_{22}\cdot\widehat{Z}_{\mathrm{w}}, \quad \widehat{T}_{12}^{\mathrm{T}} = -\widehat{Z}_{\mathrm{w}}^{-1}\cdot\widehat{T}_{21}\cdot\widehat{Z}_{\mathrm{w}}. \tag{20}$$

In the opposite direction, if the operators $\widehat{T}_{mn}$ satisfy (20) then the operators $\widehat{S}_{mn}$ are such that

$$\widehat{S}_{11}^{\mathrm{T}} = \widehat{Z}_{\mathrm{w}}^{-1}\cdot\widehat{S}_{22}\cdot\widehat{Z}_{\mathrm{w}}, \quad \widehat{S}_{12}^{\mathrm{T}} = \widehat{Z}_{\mathrm{w}}^{-1}\cdot\widehat{S}_{12}\cdot\widehat{Z}_{\mathrm{w}}, \quad \widehat{S}_{21}^{\mathrm{T}} = \widehat{Z}_{\mathrm{w}}^{-1}\cdot\widehat{S}_{21}\cdot\widehat{Z}_{\mathrm{w}}. \tag{21}$$

Eqs. (20), (21) can be rewritten in terms of the matrix elements of the corresponding operators analogously to (15).

5 Applications

In the previous sections we formulated a number of symmetry and reciprocity constraints on scattering and transfer matrices. Let us consider some consequences of these constraints.

5.1 Restrictions on optical activity

First, it is easy to see that the combination of the reciprocity and the central symmetry in a structure forbids any optical activity effects for normally incident plane waves (both in transmission and reflection into the same normally propagating mode). Indeed, from (4) and (16) we get

$$\begin{aligned} \overline{\overline{s}}_{11}(0,0) = \overline{\overline{s}}_{22}(0,0) = \overline{\overline{s}}_{11}^{\mathrm{T}}(0,0) = \overline{\overline{s}}_{22}^{\mathrm{T}}(0,0), \\ \overline{\overline{s}}_{21}(0,0) = \overline{\overline{s}}_{12}(0,0) = \overline{\overline{s}}_{21}^{\mathrm{T}}(0,0) = \overline{\overline{s}}_{12}^{\mathrm{T}}(0,0), \end{aligned} \tag{22}$$

i.e. both transmission and reflection dyadics are fully symmetric. For optical activity to occur one needs antisymmetric terms. We have the same result if the central symmetry is replaced by the planar symmetry (6). Moreover, if the structure also has rotational symmetry (7) with $m \geq 3$ then $\overline{\overline{s}}_{mn}(0,0) = s_{mn}\overline{\overline{I}}_{\mathrm{t}}$, i.e. the structure does not change the polarization state of the transmitted and reflected fields at all.

Next, let us look at the original problem of an array of quasi-planar 2D-chiral particles on top of a dielectric slab. To obtain the total transfer matrix of this system one has to multiply the transfer matrix of the grating by the transfer matrix of the slab:

$$\left(\widehat{T}_{mn}^{\mathrm{tot}}\right) = \begin{pmatrix} \widehat{T}_{11} & \widehat{T}_{12} \\ \widehat{T}_{21} & \widehat{T}_{22} \end{pmatrix} \cdot \begin{pmatrix} \widehat{\mathcal{T}}^{-1} & -\widehat{\mathcal{T}}^{-1}\cdot\widehat{\mathcal{R}} \\ \widehat{\mathcal{R}}\cdot\widehat{\mathcal{T}}^{-1} & (\widehat{\mathcal{T}} - \widehat{\mathcal{R}}\cdot\widehat{\mathcal{T}}^{-1}\cdot\widehat{\mathcal{R}}) \end{pmatrix}, \tag{23}$$

where $\widehat{\mathcal{T}}$ and $\widehat{\mathcal{R}}$ denote the transmission and reflection operators of the slab, respectively. The matrix elements of these operators are dyadics $\overline{\overline{\tau}}(\mathbf{p},\mathbf{q})$ and $\overline{\overline{\rho}}(\mathbf{p},\mathbf{q})$:

$$\overline{\overline{\tau}}(\mathbf{p},\mathbf{q}) = (2\pi)^2\,\delta(\mathbf{q}-\mathbf{p})\,\overline{\overline{\tau}}(\mathbf{q}), \quad \overline{\overline{\rho}}(\mathbf{p},\mathbf{q}) = (2\pi)^2\,\delta(\mathbf{q}-\mathbf{p})\,\overline{\overline{\rho}}(\mathbf{q}), \tag{24}$$

where $\overline{\overline{\tau}}(\mathbf{q})$ and $\overline{\overline{\rho}}(\mathbf{q})$ are dyadic transmission and reflection coefficients for a dielectric slab as functions of the transversal wave vector of the incident wave ($\mathbf{k}_t = \mathbf{q}$). The explicit expressions for them can be found in textbooks on electromagnetics.

We are interested in $\widehat{T}_{11}^{\rm tot}$ because it is connected with the transmission operator of the total system: $\widehat{S}_{21}^{\rm tot} = (\widehat{T}_{11}^{\rm tot})^{-1}$. From (23) we get

$$\widehat{T}_{11}^{\rm tot} = \widehat{T}_{11} \cdot \widehat{\mathcal{T}}^{-1} + \widehat{T}_{12} \cdot \widehat{\mathcal{R}} \cdot \widehat{\mathcal{T}}^{-1}, \tag{25}$$

or, rewriting it in terms of the matrix elements,

$$\overline{\overline{t}}_{11}^{\rm tot}(\mathbf{p},\mathbf{q}) = \overline{\overline{t}}_{11}(\mathbf{p},\mathbf{q}) \cdot \overline{\overline{\tau}}^{-1}(\mathbf{q}) + \overline{\overline{t}}_{12}(\mathbf{p},\mathbf{q}) \cdot \overline{\overline{\rho}}(\mathbf{q}) \cdot \overline{\overline{\tau}}^{-1}(\mathbf{q}). \tag{26}$$

Let us now look at the transpose of the operator $\widehat{T}_{11}^{\rm tot}$ in the case when the grating is reciprocal and has a plane of symmetry. In this case

$$(\widehat{T}_{11}^{\rm tot})^{\rm T} = \widehat{\mathcal{T}}^{-1} \cdot \widehat{Z}_{\rm w}^{-1} \cdot \widehat{T}_{11} \cdot \widehat{Z}_{\rm w} - \widehat{\mathcal{T}}^{-1} \cdot \widehat{\mathcal{R}} \cdot \widehat{Z}_{\rm w}^{-1} \cdot \widehat{T}_{21} \cdot \widehat{Z}_{\rm w}, \tag{27}$$

where we used (20) and the fact that the transmission and reflection operators of a dielectric slab are diagonal and symmetric: $\widehat{\mathcal{T}} = \widehat{\mathcal{T}}^{\rm T}$, $\widehat{\mathcal{R}} = \widehat{\mathcal{R}}^{\rm T}$. In terms of the matrix elements (27) becomes

$$\overline{\overline{t}}_{11}^{\rm tot\,T}(-\mathbf{q},-\mathbf{p}) = \overline{\overline{\tau}}^{-1}(\mathbf{p}) \cdot \left[\overline{\overline{z}}_{\rm w}^{-1}(\mathbf{p}) \cdot \overline{\overline{t}}_{11}(\mathbf{p},\mathbf{q}) - \overline{\overline{\rho}}(\mathbf{p}) \cdot \overline{\overline{z}}_{\rm w}^{-1}(\mathbf{p}) \cdot \overline{\overline{t}}_{21}(\mathbf{p},\mathbf{q})\right] \cdot \overline{\overline{z}}_{\rm w}(\mathbf{q}). \tag{28}$$

Eqs. (26) and (28) greatly simplify for the plane waves propagating along the normal:

$$\overline{\overline{t}}_{11}^{\rm tot}(0,0) = \tau^{-1}\,\overline{\overline{t}}_{11}(0,0) + \rho\,\tau^{-1}\,\overline{\overline{t}}_{12}(0,0), \tag{29}$$

$$\overline{\overline{t}}_{11}^{\rm tot\,T}(0,0) = \tau^{-1}\,\overline{\overline{t}}_{11}(0,0) - \rho\,\tau^{-1}\,\overline{\overline{t}}_{21}(0,0), \tag{30}$$

where ρ and τ are scalar reflection and transmission coefficients for a plane wave normally incident on a dielectric slab. From the last relations one can obtain an explicit formula for the antisymmetric part of $\overline{\overline{t}}_{11}^{\rm tot}(0,0)$:

$$\frac{\overline{\overline{t}}_{11}^{\rm tot}(0,0) - \overline{\overline{t}}_{11}^{\rm tot\,T}(0,0)}{2} = \rho\,\frac{\overline{\overline{t}}_{12}(0,0) + \overline{\overline{t}}_{21}(0,0)}{2\,\tau} = \rho\,\frac{\overline{\overline{t}}_{12}(0,0) - \overline{\overline{t}}_{12}^{\rm T}(0,0)}{2\,\tau}. \tag{31}$$

From here one can see that adding a dielectric slab to a reciprocal symmetric grating can indeed result in a system with a non-symmetric transmission dyadic, i.e. one can realize an optically active system this way. The higher is the reflection, the higher is the obtained optical activity. The physical reason for this is that the dielectric layer positioned on one side of the array breaks the mirror symmetry of the array, and the whole structure becomes chiral in the three-dimensional space.

Furthermore, we see that the optical activity of this kind cannot be achieved with gratings that have $\overline{\overline{t}}_{12}(0,0) + \overline{\overline{t}}_{21}(0,0) = 0$. For example, this holds for any planar (thickness tends to zero) grating made of a non-magnetic material. Indeed, for such a grating

$$\widehat{S}_{22} = \widehat{S}_{11}, \quad \widehat{S}_{21} = \widehat{S}_{12} = 1 + \widehat{S}_{11}, \tag{32}$$

therefore

$$\widehat{T}_{12} = -(1+\widehat{S}_{11})^{-1} \cdot \widehat{S}_{11} = -(1+\widehat{S}_{11}^{-1})^{-1} = -\widehat{S}_{11} \cdot (1+\widehat{S}_{11})^{-1} = -\widehat{T}_{21}. \tag{33}$$

This observation leads us to an additional conclusion that the obtained optical activity depends also on the thickness of the quasi-planar grating. Increasing the thickness (up to a certain limit related to the wavelength) one can increase the term $\overline{\overline{t}}_{12}(0,0) + \overline{\overline{t}}_{21}(0,0)$ in (31).

5.2 An array of swastika-shaped particles

As an illustrative example let us consider a square array of swastika-shaped metal particles. It was found experimentally that such a grating when backed with a dielectric slab rotates the polarization of normally incident plane waves [4]. When there is no substrate, there will be no rotation. Although this follows from our general considerations given above, let us show the same using simpler arguments.

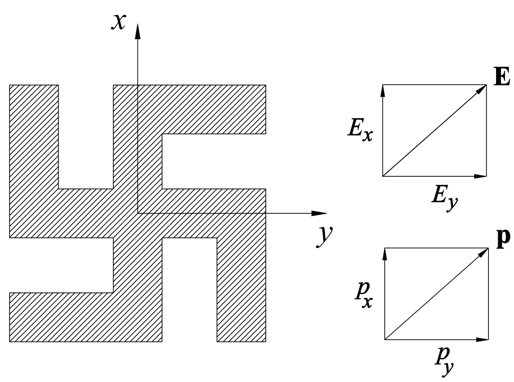

Figure 3: A swastika-shaped particle excited by an external electric field **E**. Here **p** is the induced electric dipole moment.

Consider a swastika-shaped metallic particle depicted in Fig. 3. The particle has rotational symmetry or order 4, it has a center of symmetry, and it is reciprocal. Suppose that the particle is under normal incidence of a linearly polarized plane wave with the electric field vector **E** oriented as shown in the figure. One can decompose the electric field vector of the incident wave to two components along x and y axes. Due to rotational symmetry the particle reacts equally to both these components.

The induced electric dipole moment of the particle is a sum of two components $\mathbf{p}_x$ and $\mathbf{p}_y$, and the total dipole moment $\mathbf{p} = \mathbf{p}_x + \mathbf{p}_y$ of the particle can be written as

$$\mathbf{p} = \overline{\overline{a}} \cdot \mathbf{E}, \tag{34}$$

where $\overline{\overline{a}}$ is the dyadic dipole polarizability of the particle. The rotational symmetry requires that $a_{xx} = a_{yy}$, $a_{xy} = -a_{yx}$. On the other hand, the reciprocity requires $a_{xy} = a_{yx}$, therefore $a_{xy} = a_{yx} = 0$ and $\overline{\overline{a}} = a\overline{\overline{I}}_t$, where $\overline{\overline{I}}_t$ is the two-dimensional unit dyadic. This means that the induced dipole moment is co-linear with the applied electric field.

In a square array the interaction of the particles will exhibit the same rotational symmetry, therefore for the dipole moment per unit cell of the array and the polarizability in the array

we will arrive to the same conclusions as above. The secondary field created by the array in far zone is proportional to the dipole moment per unit cell of the array. Its polarization is the same as the polarization of the induced dipole moment, and, hence, it is the same as the polarization of the incident field.

We have to note that this simplistic consideration does not include higher multipole moments, and, what is more important, it assumes that the effective induced magnetic moment per unit cell of the array is either zero (as in swastikas under normal incidence) or does not radiate in the far zone (which is the case, for instance, of omega particles under normal incidence, when the induced magnetic dipole moment is parallel to the propagation direction). It is known that one has to provide some means for magnetoelectric interactions to obtain optical activity. Adding a dielectric substrate breaks mirror symmetry of a quasi-planar array and makes optical activity possible, as we have shown in the previous section. A detailed study of how this process can be described in terms of magnetoelectric interactions is outside of the scope of this paper.

6 Conclusions

Starting from general symmetry and reciprocity (time-reversal invariance) requirements we have formulated a number of constraints on the scattering and transmission operators of arrays of quasi-planar linear reciprocal particles. We have shown that if a reciprocal array is symmetric with respect to its own plane or has a center of symmetry, then this array cannot exhibit optical activity for normally incident plane waves both in transmission or reflection. For the case of a reciprocal plane-symmetric array backed by a dielectric slab we have shown that the optical activity effects for normally incident waves occur due to reflection at the air-dielectric interface and the strength of this effect is proportional to the respective reflection coefficient. Moreover, even in the presence of a dielectric substrate, to allow for optical activity the thickness of the particles in the array should not be negligibly small.

The general restrictions on scattering and transmission operators obtained under this research apply also to grids that create several propagating modes in reflection and transmission (electrically sparse arrays producing diffraction lobes). In the last case, the restrictive relations become more complicated and they allow for asymmetries of diffraction phenomena under reversal of propagation directions. However, all these phenomena are reciprocal and agree with the fundamental time-reversal invariance of the Maxwell equations.

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